EEP/IAS 118
Andrew Dustan
Section Handout 6

## *1. Hypothesis Testing for the Mean of a Population (edit from last week)*

Hypothesis testing is pretty similar to creating a confidence interval in many ways. We are still using the fact that $\frac{\bar{x}-\mu}{s/\sqrt{n}} \sim t_{n-1}$, except now we'll actually calculate this statistic directly under the assumption of a null hypothesis about the mean of the population.

Let's go back to the MSU grade-point average example from last section. Here's the Stata summary of a random sample of students:

```
                            colGPA
-------------------------------------------------------------
      Percentiles      Smallest
 1%          2.3            2.2
 5%          2.5            2.3
10%          2.6            2.4       Obs                 141
25%          2.8            2.4       Sum of Wgt.         141

50%            3                      Mean           3.056738
                        Largest       Std. Dev.      .3723103
75%          3.3            3.8
90%          3.6            3.9       Variance        .138615
95%          3.7            3.9       Skewness       .3246205
99%          3.9              4       Kurtosis        2.59999
```

Say an administrator at MSU insists that the average university-wide GPA is 3.0. Without access to the entire student population's GPA *we can't say if he's right, but we can say if we're pretty sure he's wrong*. Following the steps from class:

**STEP 1. Define hypotheses:**

$$H_0: \mu = 3.0$$
$$H_1: \mu \neq 3.0$$

Here we're doing a two-tailed test because we don't have a good reason to think the GPA should be higher or lower than 3.0. Two-tailed tests are the most common.

Under the null hypothesis, $\mu$ is 3.0, so we assume this is true for now. If true, then $\frac{\bar{x}-3.0}{s/\sqrt{n}} \sim t_{n-1}$. Let's actually compute this "test statistic", $t_{n-1}$.

**STEP 2. Compute the test statistic:**

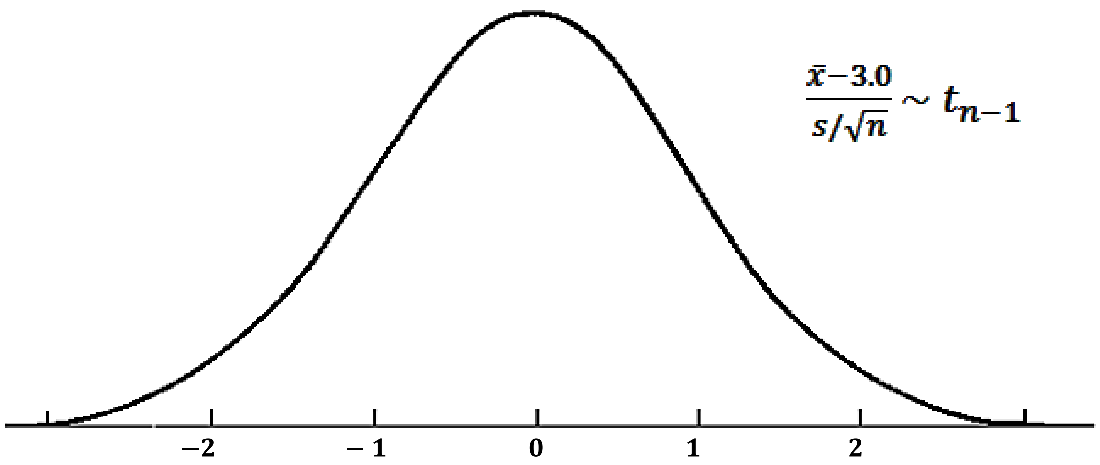
1. Get $s$: ________________________________________________

2. Get $\sqrt{n}$: ________________________________________________

3. Get n-1: ____________

4. Compute $t_{n-1}$ (this will just be a number): ____________________________________________

Now it's time to think about what $t_{n-1}$ actually means. If $\mu$ really is 3.0, then the distribution of $t_{n-1}$ looks like this:




How likely is it that I'd get my $\bar{x}$ if the true mean of GPA is 3.0? To find out, compare the t-statistic you got to the values on the axis of this graph. The farther out in a tail your t-statistic is, the more improbable your result was under the null hypothesis. If it's too improbable then we can confidently reject the notion that average GPA really is 3.0.

**STEP 3. Get the significance level of the test:**
Here we'll choose the 5% significance level, meaning we'll wrongly reject a **true** $H_0$ 5% of the time. We go to the t-table and find out what the corresponding critical value (c) is. It's about 1.98 for n-1 = 141-1 = 140.

**STEP 4. Reject the null hypothesis or fail to reject it:**

Our critical value, $c_{.05}$, is 1.98. Our t-statistic was ___________.

If our $|t_{n-1}| > 1.98$ then we reject the null hypothesis because our $\bar{x}$ was so far away from the null hypothesis of 3.0. If $|t_{n-1}| < 1.98$ then we can't reject the null hypothesis because $\bar{x}$ is close enough to the null hypothesis of 3.0 that we can't say it's wrong with enough confidence.

Did we reject $H_0$?      YES      NO

**STEP 5. Interpret:**
Either:
There is statistical evidence at the 5% significance level that the average GPA at Michigan State is different from 3.0. We have good reason to believe that the administrator was incorrect.

Or:
There is no statistical evidence at the 5% significance level that the average GPA at Michigan State is different from 3.0. We would not be confident in saying that the administrator is incorrect.

## *2. Hypothesis Testing with a Binary Variable*

Binary random variables (those taking values of 0 or 1 only) allow us to change our hypothesis testing procedure a little bit, because the null hypothesis we choose for the population *mean* also gives us a null hypothesis for the population *variance*. This is just a characteristic of binary variables—knowing the mean is sufficient to know the variance, too. Because of this, we don't need to calculate $s^2$ or use it in our test.

Example: (Based on question C.9 in Wooldridge)
Suppose that a military dictator in an unnamed country holds a plebiscite (a yes/no vote of confidence) and claims that he was supported by 65% of the voters. An NGO suspects that the dictator is lying and contracts you, a skilled econometrician, to investigate his claim. You have a small budget so you can only collect a random sample of **200 voters** in the country. From your sample of 200, you find that **115 supported** the dictator.

**STEP 1. Define hypotheses:**

$$H_0 : p = 0.65$$
$$H_1 : p < 0.65$$

Here we're doing a one-tailed test because we suspect the proportion supporting the dictator is **below** 65%. We know the variance of x under the null hypothesis is:
$var(x) = p(1-p) = 0.65(0.35) = 0.2275.$
Then for large samples, we know $var(\hat{p}) = \frac{p(1-p)}{n} = \frac{0.65(0.35)}{200} = 0.0011375.$
So under the null hypothesis, $sd(\hat{p}) = \sqrt{0.0011375} = 0.034.$

Under the null hypothesis, $\frac{\hat{p}-p}{sd(\hat{p})} \sim N(0,1)$ where $\hat{p}$ is the sample mean (the proportion of "yes" votes). Let's actually compute this "test statistic", $z$. The reason we don't use a t-statistic is that we didn't have to estimate the standard deviation of $\bar{x}$. The z-statistic is distributed the same as a t-statistic with infinite degrees of freedom.

**STEP 2. Compute the test statistic:**

1. Get $\hat{p}$, the sample mean: ______________________
2. Recall the population mean and standard deviation under the null hypothesis: _______________________
3. Compute the test statistic: $z = \frac{\hat{p}-p}{sd(\hat{p})} =$ ___________________________

**STEP 3. Get the significance level of the test:**
Here we'll choose the 5% significance level, meaning we'll wrongly reject a **true** $H_0$ 5% of the time. We go to the z-table/t-table and find out what the corresponding critical value (c) is. It's ___________.

**STEP 4. Reject the null hypothesis or fail to reject it:**

Our critical value, $c_{.05}$, is -1.64. Our z-statistic was ____________.

If our $z < -1.64$ then we reject the null hypothesis because our $\hat{p}$ was so far below the null hypothesis of 0.65. If $z > -1.64$ then we can't reject the null hypothesis because $\hat{p}$ is close enough to the null hypothesis of 0.65 that we can't say it's wrong with enough confidence.

Did we reject $H_0$? YES NO

**STEP 5. Interpret:**

## *3. Hypothesis Testing for Equality of Means between Two Populations*

Sometimes we want to see if the means of two populations are the same. We can do this without too much extra work beyond what we do for tests involving one mean.

Example (real problem, real data):
Mexico City's public schools use a competitive high school admissions system, requiring applicants to take a standardized exam and giving placement preference based on the results. Some people think that this system favors males over females because they seem to score higher on the exam. Is this true?

I have a random sample of 510 high school applicants. Here is the summary of test scores by gender:

| **Males:** | **Females:** |
|---|---|
| $n_{male} = 254$ | $n_{female} = 256$ |
| $\bar{x}_{male} = 67.15$ | $\bar{x}_{female} = 61.57$ |
| $var(x_{male}) = 417.12$ | $var(x_{female}) = 349.94$ |

**STEP 1. Define hypotheses:**
First, realize that if $\mu_{male} = \mu_{female}$, then $\Delta = \mu_{male} - \mu_{female} = 0$.

$$H_0: \Delta = 0$$
$$H_1: \Delta > 0$$

Define $d = \bar{x}_{male} - \bar{x}_{female}$. Under the null hypothesis, Δ is 0, so we assume this is true for now. If true, then $\frac{d-0}{\sqrt{\frac{s^2_{male}}{n_{male}} + \frac{s^2_{female}}{n_{female}}}} \sim t_{n_{male}+n_{female}-2}$. Let's actually compute this "test statistic", $t_{n_{male}+n_{female}-2}$.

**STEP 2. Compute the test statistic:**

1. Get d: ____________________________________________
2. Get $s^2_{male}$ and $s^2_{female}$: ______________________________________________________________
3. Get $\sqrt{\frac{s^2_{male}}{n_{male}} + \frac{s^2_{female}}{n_{female}}}$: ______________________________________________________________
4. Get $(n_{male} + n_{female} - 2)$: ______________________________
5. Compute $t_{n_{male}+n_{female}-2}$ (this will just be a number): ____________________________________

**STEP 3. Get the significance level of the test:**
Here we'll choose the 5% significance level. We go to the t-table and find out what the corresponding critical value (c) is. It's __________.

**STEP 4. Reject the null hypothesis or fail to reject it:**

Our critical value, $c_{.05}$, is 1.64. Our t-statistic was ___________.
If our $t > 1.64$ then we reject the null hypothesis because the difference in means was so high. If $t < 1.64$ then we can't reject the null hypothesis because the difference in means is close enough to the null hypothesis of 0 that we can't say it's untrue with enough confidence.

Did we reject $H_0$? YES NO

**STEP 5. Interpret:**